Quantum Communication: How quantum signals help to maintain privacy and speed things up

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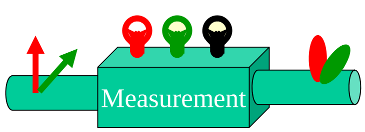
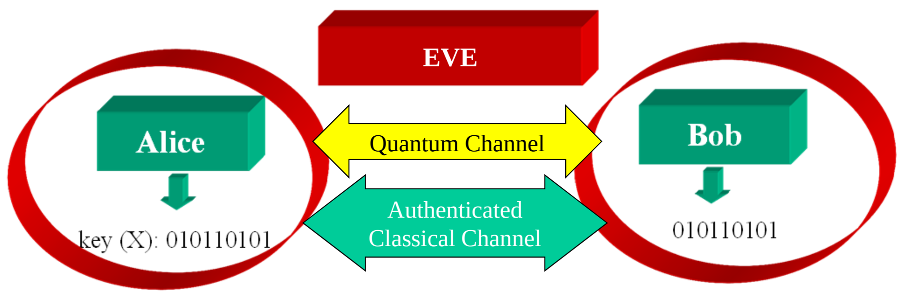
Principles of QKD in physics terms

quantum signals allow for testing of eavesdropping activity:
- Heisenberg Uncertainty principle
- back-reaction of measurement onto quantum system



eavesdroppers introduce errors
errors observed → protocol aborts
- no protection against denial-of-service attack

Quantum Key Distribution Primitive



Alice/Bob devices:

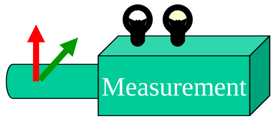
- trusted (cannot be manipulated by Eve)
- characterized (QM description known, QM believed to hold)
- secure perimeter (Eve cannot read internal status of devices)

Quantum Communication

using quantum effects in quantum communication

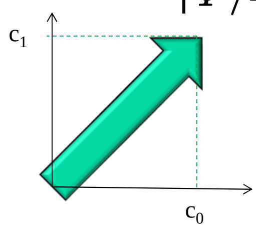
- qualitative advantage
  measurement back-reaction on signal
  ➔ quantum key distribution (cannot be achieved classically)
- quantitative advantage
  use fewer resources to accomplish a goal
  leak less information to participants (towards secure multi-party computation)

Quantum Mechanics



quantum mechanics predicts probabilities of events to happen …

$$|\Psi\rangle = \sum_i c_i |u_i\rangle$$



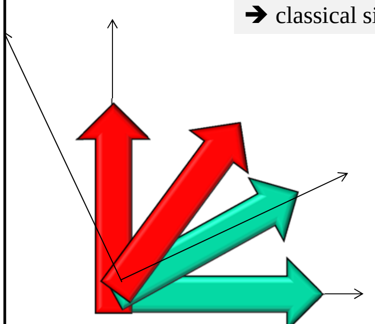
the **state of the system** is described by a
- complex unit vector $|\Psi\rangle$

The **measurement** is described by
- an orthonomal basis $\{ |u_i\rangle \}$

$$\Pr(\text{"i"}) = |c_i|^2$$

classical communication embedded in quantum mechanics

orthogonal states can be perfectly discriminated
➔ classical signals are embedded into quantum mechanical formalism



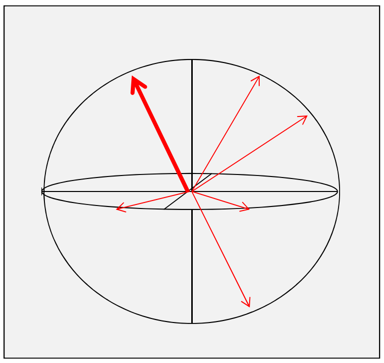
Non-orthogonal states cannot be perfectly discriminated!

$$Prob(error) \geq \frac{1}{2}\left(1 - \sqrt{1 - |\langle u|v\rangle|^2}\right)$$

but there are measurements that can unambiguously discriminate the two signals with some probability!

$$Prob(success) \leq 1 - |\langle u|v\rangle|$$

How much information can be read out of QM systems?

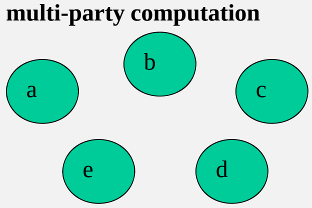


we can prepare a quantum system in an arbitrary number of different internal states!

BUT: if used in a communication context, we can recover at most $\mathbf{\log_2 d}$ number of bits about the input states

Information & Communication complexity Complexity

multi-party computation



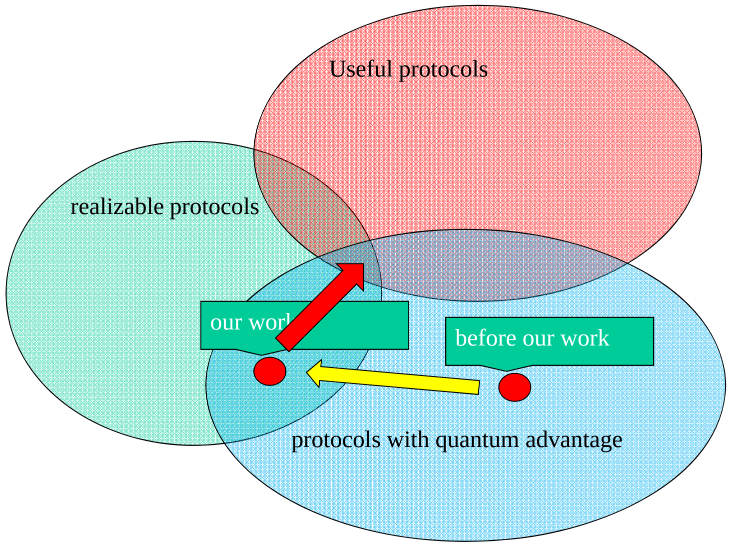
- given input: a,b,c,d,e …
- evaluate z= f(a,b,c,d,e …)

Communication Complexity:
How many signals need to be exchanged to evaluate function?

Information Complexity: (secure multi-party computation)
How much does each party learn about the input of the others?

Quantum Communication can offer better performance than classical communication

Expectation Management



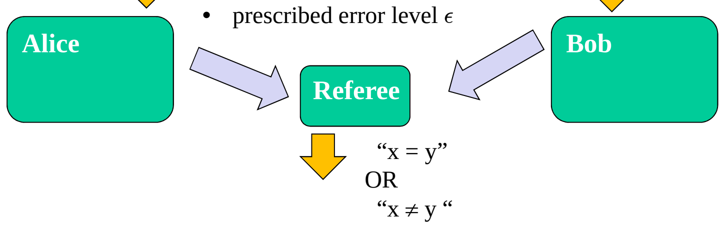
Task Description: Finger Printing

(simultaneous message passing)

$\mathbf{x} \in \{0,1\}^{\mathbf{n}}$

- one way communication only
- no shared source of randomness
- prescribed error level ϵ

$\mathbf{y} \in \{0,1\}^{\mathbf{n}}$



Exponential Gap between classical and quantum

classical $O(\sqrt{n})$ **[Ambainis, Algorithmica 16, 298 (1996)]**

quantum $O(\log_2 n)$

[Buhrman, Cleve, Watrous, de Wolf, PRL 87, 167902 (2001)]

note: If we were to give access to either

- two-way classical communication, or
- access to share randomness

➔ would also give $O(\log_2 n)$ in classical communication

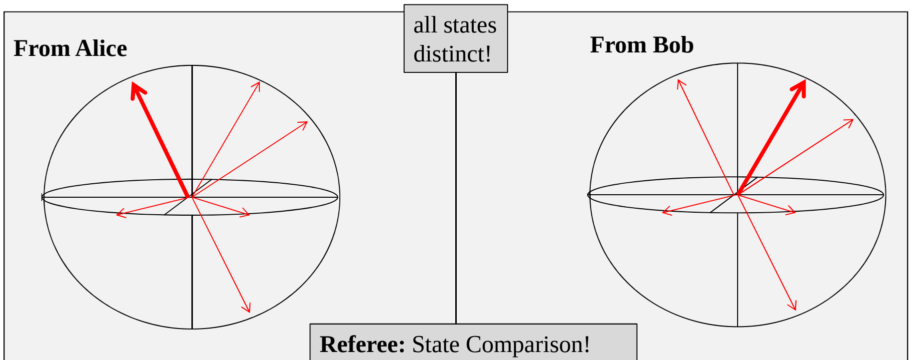
Two **different** question:

- **how many signals need to be transmitted to solve the task?**
- how much does the referee learn about the input?

Mechanism for Quantum Finger Printing

protocol encodes $\mathbf{2^n}$ states in a n dimensional Hilbert space!

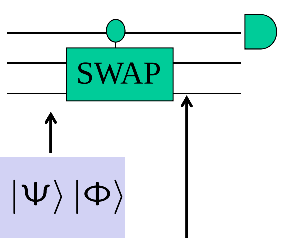
➔ highly non-orthogonal states!



Referee: State Comparison!
- are both states the same?
- not interested which state …

C-SWAP Test

Tool to give information about two states being in the same state or not …



$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, $|\Psi\rangle$, $|\Phi\rangle$

measurement in basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$$Prob('' +'') = \frac{1}{2}\left(1 + |\langle\Phi|\Psi\rangle|^2\right)$$

$$Prob('' -'') = \frac{1}{2}\left(1 - |\langle\Phi|\Psi\rangle|^2\right)$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\ |\Psi\rangle\,|\Phi\rangle$$

$$\frac{1}{\sqrt{2}}(|0\rangle\ |\Psi\rangle\,|\Phi\rangle + |1\rangle\ |\Phi\rangle\ |\Psi\rangle)$$

	Equal input	Unequal input
'same' (+)	1	$\left[\frac{1}{2}\left(1 + |\langle\Phi|\Psi\rangle|^2\right)\right]^n$
'different' (-)	0	$1 - \left[\frac{1}{2}\left(1 + |\langle\Phi|\Psi\rangle|^2\right)\right]^n$

➔ 0 for n $\to \infty$

➔ 1for n $\to \infty$

If n repetitions allowed
➔ can quickly reduce

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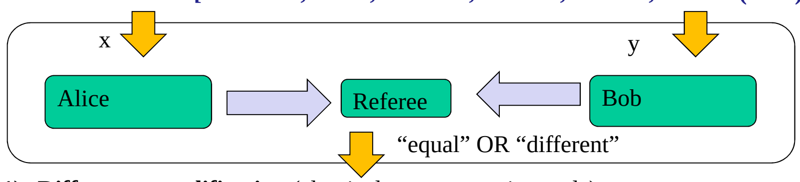
Quantum Finger Printing Protocol

[Buhrman, Cleve, Watrous, de Wolf, PRL 87, 167902 (2001)]



1) **Difference amplification** (classical error correction code)
   x → E(x)
   n bits → m > n bits
   Hamming weight d(E(x), E(x')) > (1-δ) m

(we will later on use m = 3 n and δ = 0.92)

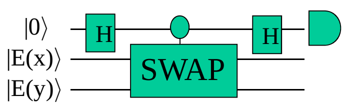
→ **one bit difference**
→ **8% error difference**

2) Alice, Bob: **Quantum encoding**

$$E(x) \rightarrow |E(x)\rangle := \frac{1}{\sqrt{m}} \sum_{i=1}^{m} (-1)^{E(x)_i} |i\rangle$$

 # qubits: log m

3) Referee: **Conditional-SWAP test**



	Equal input	Unequal input
'same'	1	< ½(1+δ^2)
'different'	0	> ½(1-δ^2)

4) **k-fold repetition** to reduce errors < ϵ [require repetition: k = O(log 1/ϵ)]

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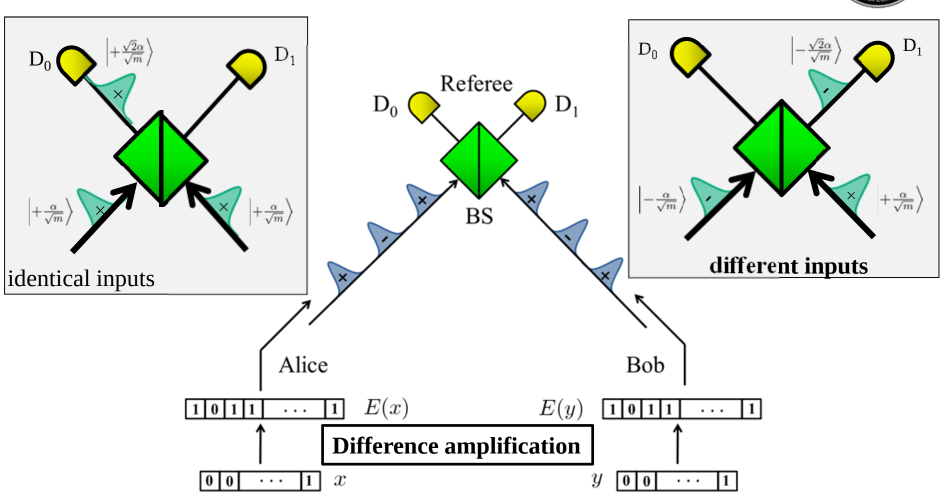
Coherent-state Protocol

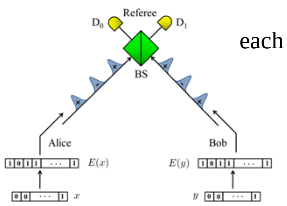
[Arrazola and Lütkenhaus, Phys. Rev A 89, 062305 (2014)]



overall identical inputs: only detector D_0 clicks
some differences: some D_0 clicks, some D_1 clicks

occurrence of D_1 detector clicks
→ "overall different"
→ else: "overall identical"

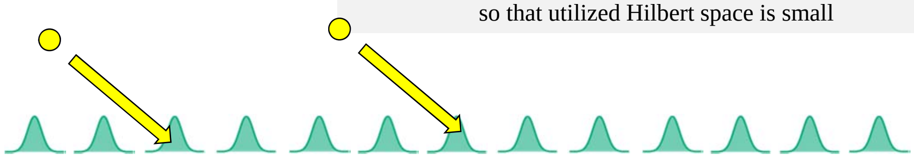
Resource counting



each pulse $\left|+\frac{\alpha}{\sqrt{m}}\right\rangle$

➔ $\Pr(\text{click}) = 1 - e^{-2\frac{|\alpha^2|}{m}} \approx 2\frac{|\alpha^2|}{m}$

make overall mean photon number $|\alpha|^2$
➔ sufficiently large
such that at least one click if difference exists
➔ sufficiently low
so that utilized Hilbert space is small



1 photon in m modes ➔ dimension Hilbert space m, ➔ ***log m*** qubits

N photons in m modes ➔ dim is $\binom{N+m-1}{m-1} \approx m^N$ ➔ ***O(N log m)*** qubits

Experimental realities

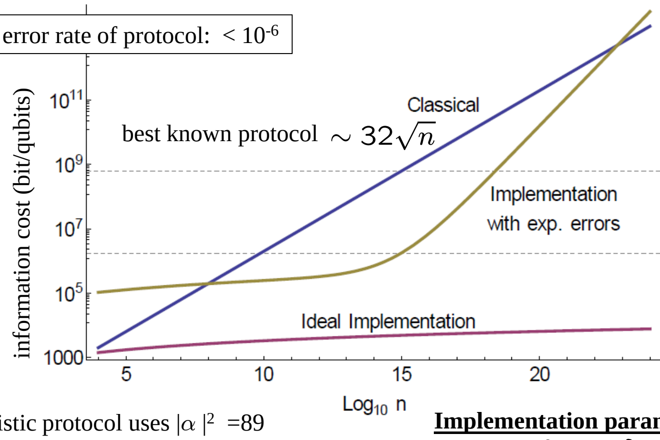
loss between sources and referee?
➔ simply increase mean photon number to compensate loss
➔ does not affect scaling of resources!

dark count in detectors?
➔ set optimal threshold scheme to decide ‘overall identical’ or ‘overall different’
➔ will affect scaling for larger input size states:need to maintain signal/noise ratio

mode matching on beam splitter?
➔ uses again optimal threshold scheme to discriminate ‘identical/different’
➔ does not affect scaling, as errors are proportional to signal

Simulation optical system example of combined effects

target error rate of protocol: $< 10^{-6}$



- ➔ idealistic protocol uses $|\alpha|^2 = 89$
- ➔ realistic protocol uses $|\alpha|^2 = 6651$
  - ➔ starting at $n = 10^{13}$ one needs to increase $|\alpha|^2$ to balance increasing dark count effects

Implementation parameters:

error amplification $\delta = 0.92$ [m = 3 n]

$\eta = 0.1$ ➔ **90% loss!!**

dark count probability $d_B = 4 \times 10^{-9}$

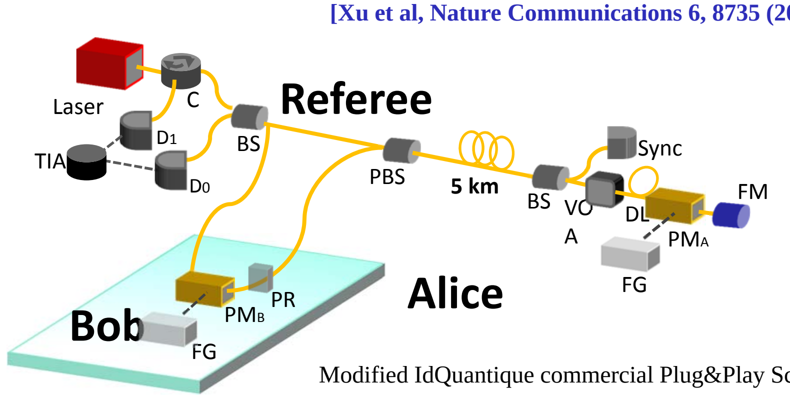
visibility v = 0.98

Implementation

Experimental Quantum Fingerprinting

Feihu Xu,[1,2,*] Juan Miguel Arrazola,[3,*] Kejin Wei,[1,4] Wenyuan Wang,[1,5] Pablo Palacios-Avila,[3,6] Chen Feng,[7] Shihan Sajeed,[8] Norbert Lütkenhaus,[3,†] and Hoi-Kwong Lo[1,‡]

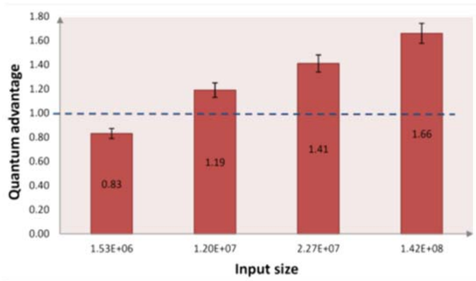
[Xu et al, Nature Communications 6, 8735 (2015)]



Modified IdQuantique commercial Plug&Play Scheme

Experimental Results

Quantum advantage: $\gamma = \frac{C}{Q}$

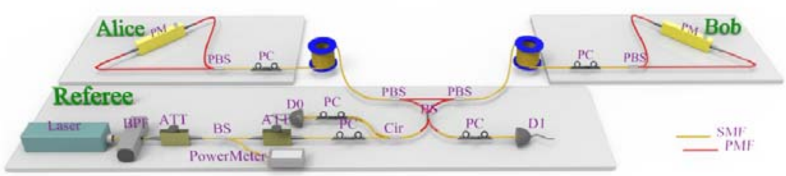


d_{det}= 3.5 x 10^{-6}
η_{det} = 20%
clockrate 5 MHz
5km distance Alice/Referee to Bob

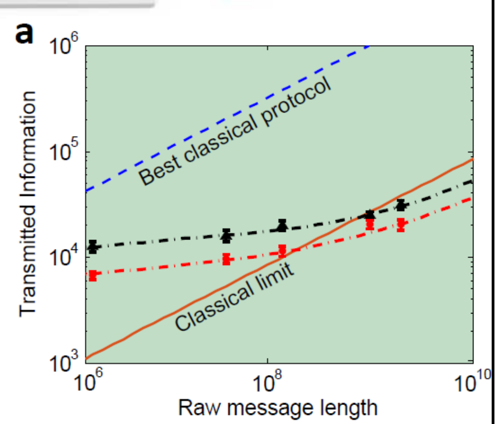
Note: We use roughly 7,000 photons for input size of 10^8!

Another experimental realization …

[Guan, Zhang, Pan et al, Phys. Rev. Lett. 116, 240502 (2016)]



beats not only best known classical protocol, but also best known bound on any classical protocol



Will this convince an optical communication engineer?

[Phys. Rev A 90, 042335 (2014)]

classical: number of bits $O(\sqrt{n})$

Our quantum implementation: number of pulses: n
Dimension: log n

BUT:

encoding has constant energy (photon number)

➔ number of photons in the channel dramatically decreased

- reduced cross-talk in fiber
- fewer detection clicks expected ➔ faster clock rates???

ALSO

does not require time resolution in detector!
Accumulation of photons would just be fine
➔ allows higher clock rate

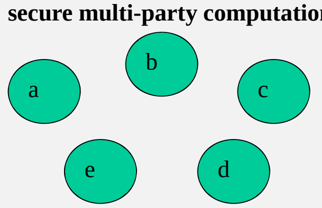
AND

leaks only O(log n) bits about strings x, y to referee
➔ Information Complexity
see our paper **[Arrazola, Touchette, arXiv:1607.07516]**

Information Complexity

How much does each party learn about the input of the others?

secure multi-party computation



- given input: a,b,c,d,e …
- evaluate z= f(a,b,c,d,e …)
- so that all parties know z and their own input
- but nothing else

cannot be achieved exactly
[Buhrman, Christandl, Schaffner, | Phys. Rev. Lett. 109, 160501 (2012)]

For Quantum Fingerprinting:

- equality function
- communication constraints: one-way, no shared randomness
- Bound on classical protocol: $O(\sqrt{n})$
  (exact expression known!)

[Arrazola, Touchette, arXiv:1607.07516]

➔ our quantum optical protocol can beat that!

The story continues …

Encoding scheme can be used to address

- hidden matching protocol

  (needs programmable mode switching)

- can be translated to other communication complexity protocols maintaining quantum advantage **[J.M. Arrazola, N. L, Phys. Rev. A 90, 042335 (2015)]]**

- can be used by other quantum protocols (quantum retrieval games)

  [Arrazola, Karasamanis, NL, Phys. Rev. A 93, 062311 (2016)]

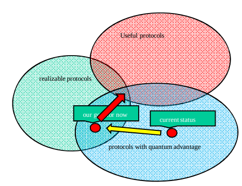
appointment scheduling problem

- has quadratic quantum advantage
- has an optical implementation shuttling laser pulses for and back
- is still very susceptible to coupling losses

Summary

- There is a path to implement **scalable quantum communication complexity protocols!**

  ➔ think about other useful protocols



- advantage in use of **Hilbert space dimensions**, **number of photons** used

- entry into world **information complexity protocols**

  (direction of secure multi-party computations)